## The Curve Graph of the 5-Punctured Sphere

## Our Project

Hensel, Przytycki, and Webb proved the hyperbolicity constant of the curve graph of a surface is $\leq 17$, and we show it is $>1$ in the case of the 5 -punctured sphere.

## Hyperbolicity

Hyperbolic space comes in many forms: trees, the hyperbolic plane, curve graphs of surfaces, etc.



Euclidean space is not hyperbolic but a tree is

## Centered Triangles

A triangle in $C(S)$ is $\delta$-centered if there exists a vertex that is at most $\delta$ away from each side. If all triangles are $\delta$-centered, then we say $C(S)$ is $\delta$-hyperbolic.


0-centered triangle


1-centered triangle

## Curve Graph C(S)

The curve graph $C(S)$ of a surface $S$ is the graph where vertices are curves and the edges represent disjointedness.


## Not 1-Centered

Lemma: A geodesic triangle is not 1-centered if the following hold:
1.) $\min \{d(b, \alpha): \alpha \in S\} \geq 3 ;$
2.) $\min \left\{\mathrm{d}(c, \alpha): \alpha \in \mathrm{S}_{2}\right\} \geq 3$;


## Checking Condition 2



## $C\left(\Sigma_{0,5}\right)$ is not 1-hyperbolic

Theorem (Aurin-Thornburgh): The
following curves form a geodesic triangle that is not 1-centered.


Future Work
Extending our result to $C\left(\Sigma_{0, n}\right)$ for all $\mathrm{p} \geq 6$ and possibly to surfaces with genus. Can we do something similar in the arc graph?

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