The Curve Graph of the 5-Punctured Sphere

Sami Aurin¹ and Darrion Thornburgh²

Our Project

Hensel, Przytycki, and Webb proved the hyperbolicity constant of the curve graph of a surface is ≤ 17 , and we show it is > 1 in the case of the 5-punctured sphere.

Hyperbolicity

Hyperbolic space comes in many forms: trees, the hyperbolic plane, curve graphs of surfaces, etc.





Euclidean space is not hyperbolic but a tree is

Centered Triangles

A triangle in C(S) is δ -centered if there exists a vertex that is at most δ away from each side. If all triangles are δ -centered, then we say C(S) is δ -hyperbolic.



0-centered triangle



1-centered triangle

Mentors: Wade Bloomquist¹ and Dan Margalit¹





Curve Graph C(S)

The **curve graph** *C*(*S*) of a surface *S* is the graph where vertices are curves and the edges represent disjointedness.





Not 1-Centered

Lemma: A geodesic triangle is not 1-centered if the following hold:

1.) $\min\{d(b,\alpha) : \alpha \in S_1\} \ge 3;$ 2.) min{d(c,α) : $\alpha \in S_2^{-1}$ } \geq 3;

Checking Condition 2





 $d(c,a) \geq 3$

 $d(c,x) \ge 3$

 $d(c,y) \ge 3 \blacksquare$





We would like to thank our mentors Dr. Wade Bloomquist and Dr. Dan Margalit. We would also like to thank the NSF for funding this research.





$C(\Sigma_{05})$ is not 1-hyperbolic

Theorem (Aurin-Thornburgh): The following curves form a geodesic triangle that is



Future Work

Extending our result to $C(\Sigma_{0,p})$ for all $p \ge 6$ and possibly to surfaces with genus. Can we do something similar in the arc graph?

Acknowledgments

1: Georgia Institute of Technology



2: Bard College