The Curve Graph of the 5-Punctured Sphere
Sami Aurin¹ and Darrion Thornburgh²  
Mentors: Wade Bloomquist¹ and Dan Margalit¹

Our Project
Hensel, Przytycki, and Webb proved the hyperbolicity constant of the curve graph of a surface is ≤ 17, and we show it is > 1 in the case of the 5-punctured sphere.

Hyperbolicity
Hyperbolic space comes in many forms: trees, the hyperbolic plane, curve graphs of surfaces, etc.

Centered Triangles
A triangle in C(S) is δ-centered if there exists a vertex that is at most δ away from each side. If all triangles are δ-centered, then we say C(S) is δ-hyperbolic.

Theorem (Aurin-Thornburgh): The following curves form a geodesic triangle that is not 1-centered.

Lemma: A geodesic triangle is not 1-centered if the following hold:
1.) min{d(b,a) : α ∈ S₁} ≥ 3;
2.) min{d(c,α) : α ∈ S₂} ≥ 3;
...

Future Work
Extending our result to C(Σₚ) for all p ≥ 6 and possibly to surfaces with genus. Can we do something similar in the arc graph?

Acknowledgments
We would like to thank our mentors Dr. Wade Bloomquist and Dr. Dan Margalit. We would also like to thank the NSF for funding this research.