The Non-Simple Arc Graph
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Arc graph: vertices are arcs and edges are disjointness.

Goal: Non-simple arc graph is connected with infinite diameter.

Connected

Theorem [Gardiner–McPoyle]: The non-simple arc graph is connected.

Key tool: unicorn arcs and unicorn paths.

Idea of proof: Consecutive unicorn arcs still adjacent in non-simple arc graph → use unicorn paths to connect any vertices in our non-simple arc graph.

Question: What is adjacency in the non-simple arc graph?

New condition: Arcs $a, b$ are adjacent if $i(a, b) \leq i(a, a) + i(b, b)$.

2 ≤ 8 + 0

Non-Simple Arcs

Used by Maryam Mirzakhani in her research!

Arrows can now self-intersect.

Infinite Diameter

Theorem (in progress): The non-simple arc graph has infinite diameter.

Consider the inclusion:

$A(S) \rightarrow NSA(S)$

Want: distances are coarsely preserved. Take a dist $D$ path in NSA(S) between simple arcs $a$ and $b$:

Step 1: modify path to have no consecutive non-simple arcs

Lemma (in progress): For any two adj. non-simple arcs, there is a simple arc adj. to both.

Step 2: replace all non-simple arcs with simple

What’s the largest distance $D’$ this path can be?

Lemma (in progress): Non-simple distance $2 \Rightarrow$ at most distance 3 in simple arc graph.

For example, this would mean that we can’t find a non-simple arc that makes this distance 4 pair distance 2.

If we prove this, then $D’ \leq 3D$.

The arc graph is infinite diameter, so the non-simple arc graph must be as well.

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