The Simple Arc Graph

Arc graph: vertices are arcs and edges are disjointness.



Non-Simple Arcs

Used by Maryam Mirzakhani in her research!





Arcs can now self-intersect.

Question:

What is adjacency in the non-simple arc graph?

New condition: Arcs a, b are adjacent if $i(a,b) \le i(a,a) + i(b,b).$





The Non-Simple Arc Graph

Sophie Gardiner (Haverford College) and Alyssa McPoyle (Bowdoin College) Mentors: Dr. Wade Bloomquist and Dr. Dan Margalit

Goal:

Non-simple arc graph is connected with infinite diameter.

Connected

Theorem [Gardiner–McPoyle]: The non-simple arc graph is connected.

Key tool: unicorn arcs and unicorn paths.



Idea of proof: Consecutive unicorn arcs still adjacent in non-simple arc graph \rightarrow use unicorn paths to connect any vertices in our non-simple arc graph.



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Theorem (in progress): The non-simple arc graph has infinite diameter.

Consider the inclusion:



Want: distances are coarsely preserved. Take a dist D path in NSA(S) between simple arcs *a* and *b*:







For example, this would mean that we can't find a non-simple arc that makes this distance 4 pair distance 2.

If we prove this, then $D' \leq 3D$.

The arc graph is infinite diameter, so the non-simple arc graph must be as well.

Infinite Diameter

 $\mathcal{A}(S) \to \mathcal{NSA}(S)$