Persistent Legendrian Contact Homology

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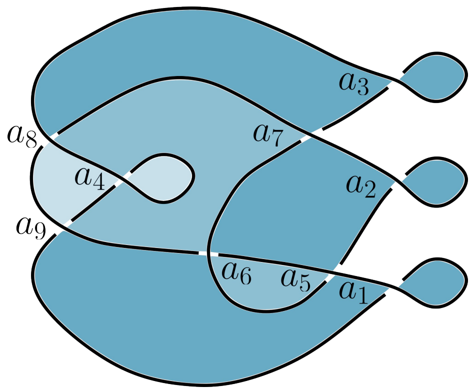
Background

Definition: A Legendrian knot in $\mathbb{R}^3$ is a simple closed curve satisfying the differential equation $dz\text{-}ydx=0$.

Goal: Distinguish Legendrian knots by finding invariants that are preserved under (Legendrian) isotopy.

Method: Assign "heights" to each crossing in the xy-projection via our flooding algorithm and compute persistent homology.

Our Flooding Algorithm



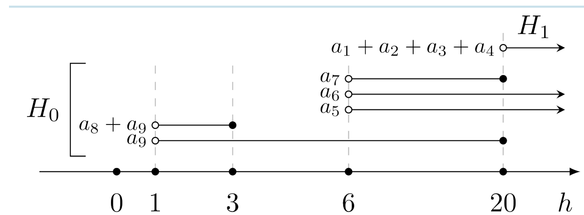
Theorem (B-C-M)
Every Legendrian knot is isotopic to a Legendrian knot whose algorithm terminates.

Tiers	1st	2nd	3rd	4th
Crossings	$a_1\ a_2\ a_3$	$a_5\ a_6\ a_7$	a_4	$a_8\ a_9$
Heights	20	6	3	1

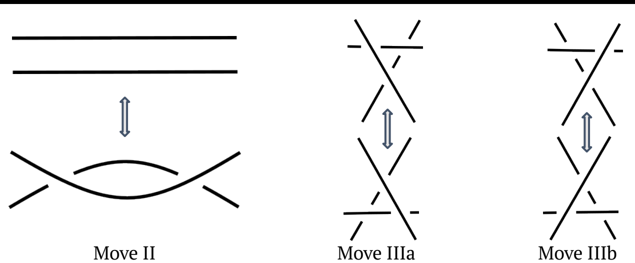
Persistent Homology

$$
\begin{array}{lccccccc}
 & C_2 & & C_1 & & C_0 & & C_{-1} \\
h=1 & 0 & \xrightarrow{\partial^\epsilon} & 0 & \xrightarrow{\partial^\epsilon} & \mathbb{Z}_2\langle a_8, a_9\rangle & \xrightarrow{\partial^\epsilon} & 0 \\
 & \cap & & \cap & & \cap & & \cap \\
h=3 & 0 & \xrightarrow{\partial^\epsilon} & \mathbb{Z}_2\langle a_4\rangle & \xrightarrow{\partial^\epsilon} & \mathbb{Z}_2\langle a_8, a_9\rangle & \xrightarrow{\partial^\epsilon} & 0 \\
 & \cap & & \cap & & \cap & & \cap \\
h=6 & 0 & \xrightarrow{\partial^\epsilon} & \mathbb{Z}_2\langle a_4\rangle & \xrightarrow{\partial^\epsilon} & \mathbb{Z}_2\langle a_5, a_6, a_7, a_8, a_9\rangle & \xrightarrow{\partial^\epsilon} & 0 \\
 & \cap & & \cap & & \cap & & \cap \\
h=20 & 0 & \xrightarrow{\partial^\epsilon} & \mathbb{Z}_2\langle a_1, a_2, a_3, a_4\rangle & \xrightarrow{\partial^\epsilon} & \mathbb{Z}_2\langle a_5, a_6, a_7, a_8, a_9\rangle & \xrightarrow{\partial^\epsilon} & 0
\end{array}
$$

From a Legendrian knot we get a chain complex (above) at each height allowing us calculate persistent homology (below).

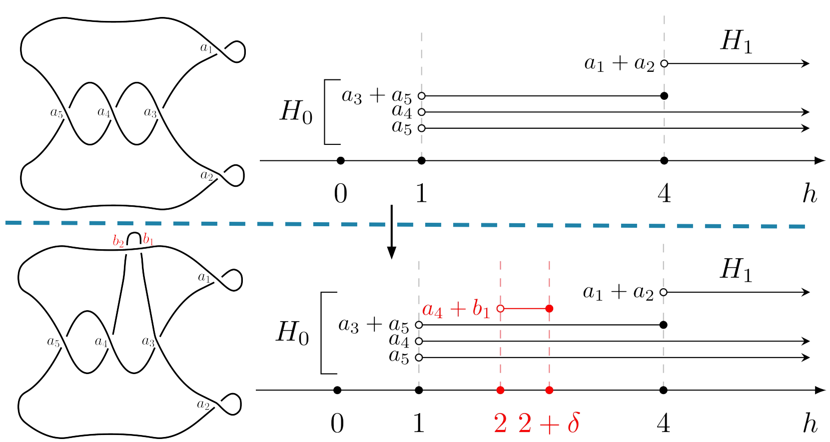


Legendrian Reidemeister Moves



Theorem
Two Legendrian knots are isotopic only if they are related by a sequence of the above Reidemeister moves in their xy-projection.

Barcode Analysis



Theorem (Work in Progress)
For any valid Legendrian Reidemeister move, there exist height assignments such that the persistent homology *barcodes* of the knot before and after the move are 2δ-interleaved.