Cooperative Energy Dynamics of a Vibro-Impact System

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**Introduction**

- Vibro-Impact Systems (VIS) can be used to convert ambient vibrations into electrical energy and store vibrational energy in the system. Our VIS is a capsule containing a ball that hits capacitors on either end to generate energy and absorb energy from the capsule.
- Targeted Energy Transfer (TET) in this VIS is the transfer of vibrational energy from the environment into controllable, dissipative sinks, such as the ball.
- Energy Harvesting (EH) in this VIS refers to the generation of energy through collisions by deforming capacitors on either end of the capsule.

**Research Goals**

- Better understand the behavior of TET and EH in the same parameter ranges for the system.
- Determine parameter ranges where TET and EH behave asymmetrically.
- Identify parameter ranges that are stable and productive for both TET and EH.

**System Dynamics**

We define \( x(t) = x_1(t) - x_2(t) \), velocities accordingly. The ball-and-capsule plots show the absolute position of the ball over time, as well as the absolute positions of each end of the capsule. We use fourth order Runge-Kutta with a bisection algorithm to numerically simulate a system dynamics for \( \omega = 7 \pi \text{rad/s} \) and \( \beta = 22 \pi \text{rad} \).

**Energy Heuristics**

We plot the behavior of the system in terms of relative impact velocities and the resulting TET and EH across various inclinations. Away from resonance, 1-to-1 impact behavior seems to occur, and TET and EH appear jointly high. Near resonance, more complex impact patterns are observed, and the measures appear inversely related.

**Fixed parameters**

<table>
<thead>
<tr>
<th>System Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capsule Mass (M)</td>
<td>1.0 kg</td>
<td>m</td>
</tr>
<tr>
<td>Spring Constant (k)</td>
<td>25e5 N/m</td>
<td>N/m</td>
</tr>
<tr>
<td>Phase Offset (( \phi ))</td>
<td>0.8</td>
<td>( \text{rad} )</td>
</tr>
<tr>
<td>Horizontal Amplitude (A)</td>
<td>5</td>
<td>m</td>
</tr>
<tr>
<td>Damping Coefficient (c)</td>
<td>0.1245 Ns/m</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Ball Mass (m)</td>
<td>0.0254 kg</td>
<td>kg</td>
</tr>
</tbody>
</table>

**Interpolated TET/EH Behavior**

We extend our analysis by plotting TET and EH measures across varying forcing frequencies and inclinations and interpolating the results, to better understand the effects of resonance on the system. Resonance occurs near \( \omega = 5 \pi \text{rad/s} \), equivalent to 2.5 Hz.

**Semi-Analytical Maps**

We use discrete maps between collision states of the system to calculate fixed points and predict 1-to-1 impact motion. In the future, we can then linearize the map to prove stability and locate stable 1-to-1 impact solutions.

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**References**
