Improving Energy Harvesting Using a Coupled Vibro-Impact System

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Introduction

- A vibro-impact energy harvester consists of a ball which moves through a hollow capsule driven by harmonic forcing [1].
- When the ball hits the edge, it deforms a dielectric-elastomer membrane, generating excess electrical energy that can be used externally [1].
- We explore a pair of energy harvesters **coupled** with a linear spring and damper to understand whether they improve energy harvesting over uncoupled harvesters.

Lumped Element Model



Figure 1. Lumped element diagram of the coupled Whenever $|q_{iC} - q_{iB}| = s/2$ energy harvesters.

Governing Equations

 $Mq_{1C}'' + c\left(2q_{1C}' - q_{2C}'\right) + k_1q_{1C}$ $+k_2(q_{1C} - q_{2C}) = \|\hat{F}\|\cos(\omega\tau + \varphi)\|$

$$Mq_{2C}'' + c \left(q_{2C}' - q_{1C}'\right) + k_2(q_{2C} - q_{1C}) = \|\hat{F}\| \cos(\omega\tau + q_{1C}')$$

 $q_{1B}'' = q_{2B}'' = -g\sin\beta.$ (at impact),

$$Mq_{iC}^{\prime +} + mq_{iB}^{\prime +} = Mq_{iC}^{\prime -} + mq_{iB}^{\prime -}$$
$$q_{iC}^{\prime +} - q_{iB}^{\prime +} = -r\left(q_{iC}^{\prime -} - q_{iB}^{\prime -}\right).$$

Numerical Methods

- We simulate the equations above using a fourth-order Runge-Kutta method.
- We find the ball and capsule velocities before each impact and use them to compute energy harvested per collision for the coupled harvesters and a decoupled harvester with the same natural frequency.

Resonance Effects

- A coupled oscillator system has **two natural frequencies**. We explore near **resonance**, where one of the system's natural frequencies is equal to the forcing frequency.
- At first resonance, higher impact speeds occur at low forcing amplitudes.
- Higher impact speeds occur at high forcing amplitudes at second resonance.
- We conjecture that as the forcing amplitude increases, the **optimal resonant regime** for energy harvesting shifts from first resonance to second resonance.

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Analytical Methods

- We construct two **discrete-time maps** to analytically determine the state of the system at consecutive impacts on opposite sides of a capsule.
- In the 1:1 regime, stable **fixed points** of the composition of these maps reveal pre-impact velocities.



Steady-State Dynamics

Figure 2. Plots depicting an example of the system's long-time behavior.



Bifurcation Diagrams

Figure 3. Bifurcation diagrams for the first (top) and second (bottom) harvesters, varying the forcing amplitude (left) and spring constants (right) at second resonance.

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Energy Harvesting Heatmaps



Figure 4. Voltage harvested per impact (top) and improvement over a decoupled harvester (bottom), varying the forcing amplitude (left) and spring constants (right) near second resonance (gold line). The coupled harvesters outperform the decoupled one in blue regions, and the decoupled one performs better in red regions.

Conclusions

- In some parameter regimes, coupled energy harvesters output more voltage than previous designs.
- Excessive coupling **decreases** the amount of electrical energy harvested.
- More energy is harvested in periodic regimes (especially 1:1) than chaotic ones.

Future Work

- Finding additional solutions analytically, determining their stability, and expanding the analytics beyond the 1:1 regime.
- Verifying numerical results in an experimental setting.
- Exploring networks of several energy harvesters and whether adding more coupled energy harvesters might further improve energy output.

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References

[1] L. Serdukova, R. Kuske, and D. Yurchenko, Stability and bifurcation analysis of the period-t motion of a vibroimpact energy harvester, Nonlinear Dynamics, 98 (2019), pp. 1807-1819.