Improving Energy Harvesting Using a Coupled Vibro-Impact System

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Introduction

- A vibro-impact energy harvester consists of a ball which moves through a hollow capsule driven by harmonic forcing [1].
- When the ball hits the edge, it deforms a dielectric-elastomeric membrane, generating excess electrical energy that can be used externally [1].
- We explore a pair of energy harvesters coupled with a linear spring and damper to understand whether they improve energy harvesting over uncoupled harvesters.

Lumped Element Model

Figure 1. Lumped element diagram of the coupled energy harvesters.

Governing Equations

\[
Mq''_c + c(q'_{c} - q'_{B}) + k(q_{c} - q_{B}) = [F(t)]\cos(\omega t + \phi)
\]

Numerical Methods

- We simulate the equations above using a fourth-order Runge-Kutta method.
- We find the ball and capsule velocities before each impact and use them to compute energy harvested per collision for the coupled harvesters and a decoupled harvester with the same natural frequency.

Resonance Effects

- A coupled oscillator system has two natural frequencies. We explore near resonance, where one of the system's natural frequencies is equal to the forcing frequency.
- At first resonance, higher impact speeds occur at low forcing amplitudes.
- Higher impact speeds occur at high forcing amplitudes at second resonance.
- We conjecture that as the forcing amplitude increases, the optimal resonant regime for energy harvesting shifts from first resonance to second resonance.

Analytical Methods

- We construct two discrete-time maps to analytically determine the state of the system at consecutive impacts on opposite sides of a capsule.
- In the 1:1 regime, stable fixed points of the composition of these maps reveal pre-impact velocities.

Steady-State Dynamics

Bifurcation Diagrams

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Future Work

- Finding additional solutions analytically, determining their stability, and expanding the analytics beyond the 1:1 regime.
- Verifying numerical results in an experimental setting.
- Exploring networks of several energy harvesters and whether adding more coupled energy harvesters might further improve energy output.

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References