Deck Reconstruction for Restricted Word Classes

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Introduction

Words & Subword Counts

A binary word is $x = x_1x_2\cdots x_n$ with $x_i \in \{0,1\}$. $N(w,x)$ counts the copies of w occurring as a subsequence of x.

Example

$$x = 0111001 \quad N(010,x) = 6$$

The k-Deck

Define the k-deck of a word as every length-k subword paired with its count:

$$\mathcal{D}_k(x) = \{(w, N(w,x)) : w \in \{0,1\}^k\}.$$

For $x = 0111001$,

w	000	001	010	011	100	101	110	111
$N(w,x)$	1	3	6	6	3	6	6	4

Deck Equivalence

$$x \overset{k}{\sim} y \text{ denotes } \mathcal{D}_k(x) = \mathcal{D}_k(y).$$

If two words share the same k-deck, they cannot be uniquely reconstructed.

Under what conditions can we determine a word given only its k-deck?

Example

$$0110 \overset{2}{\sim} 1001 \quad 0110 \overset{3}{\not\sim} 1001$$

The Reconstruction Problem

The least deck order reconstructing every length-n word is defined as

$$f(n) := \min\{k : \nexists\, x \neq y \in \{0,1\}^n,\ x \overset{k}{\sim} y\}.$$

Dudík–Schulman (Lower Bound), and Krasikov–Roditty (Upper bound) give:

$$\exp\big(\Omega(\sqrt{\log n})\big) \leq f(n) \leq O(\sqrt{n}).$$

Main Takeaway

We study the analogous reconstruction threshold for specific families of words and obtain bounds for these thresholds that are length independent.

Polynomial Support Words

The support of a word x is given by

$$\mathrm{supp}(x) := \{i : x_i = 1\}.$$

For example:

$$\mathrm{supp}(01100101) = \{2,3,6,8\}.$$

We define the class

$$\mathcal{P}_d := \{x : \mathrm{supp}(x) \text{ is given by a monotonic degree-}d \text{ polynomial}\}.$$

Example

$$100100001 \in \mathcal{P}_2 \quad p(t) = t^2 \quad N(1,x) = 3$$

We study the threshold

$$f(\mathcal{P}_d) := \min\{k : \nexists x \in \mathcal{P}_d, y \in \{0,1\}^*, x \overset{k}{\sim} y, x \neq y\}.$$

Our Result

For every $d \geq 0$,

$$f(\mathcal{P}_d) \leq d + 2.$$

Example

Since the support of the following word is given by $p(t) = 1 + 3t$, it is 3-reconstructible:

$$000100100100