Introduction

Let $T^2 = \mathbb{R}^2 / 2\pi\mathbb{Z}^2$ be the two-dimensional torus. The map $S : T^2 \to T^2$ defined as $S(v) = S_0 + f(v) \mod 2\pi$, where $S_0 = \left(\frac{1}{2}, \frac{1}{2}\right)$, is called Arnold’s Kitten Map. The matrix $S$ has eigenvalues $\lambda_\pm = \pm 2\sqrt{\epsilon}$, $S$ is an elementary example of an Anosov Diffeomorphism. A precise definition is given in [Mathematical Equations].

Main Goal

It is well known that the unstable set under an Anosov diffeomorphism $f : M \to M$ given by $W^u(f) = \{y \in M : dF^n f^{-n}(y) \to 0 \text{ as } n \to \infty\}$ is a manifold whose regularity is as high as that of $f$. However, it is very hard to get a description of its tangent space. We will give an explicit characterization of the tangent bundle of the stable set $S$ under a small perturbation.

Perturbation of Arnold’s Cat Map, Conjugation

We consider a small perturbation $S$ of $S_0$ on $T^2$ given by $S_0(v) = S_0(v) - f(v) \mod 2\pi$, where $f$ is a trigonometric polynomial. Then, there exists a unique homeomorphism $H$, analytic in $\epsilon$, such that $H(0) = S_0$, and $H^\epsilon$ is on $T^2$. An expression for $H$ can be found in [Mathematical Equations].

Conclusions

Even though the derivative of $H_\epsilon$ does not exist, there is most likely still a way to find the vector field of the unstable manifold of $S$. The above result was derived by dividing the derivative of $H_\epsilon$ by its length, expressing the values in terms of trees, and then seeing that the values of those trees “cancel out”.

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References


Further Directions

- For the 3d case, we look at the matrix $S_0 = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$
- The matrix $S_0$ has three eigenvalues, 2 of which are greater than 1, and one whose absolute value is less than 1. The equation for $H_\epsilon$ is entirely similar to the 2d case (See: Mathematical Equations).
- However, the cancellation is much more complicated, and our methods do not work as well.