# Dynamics of a Vibro-Impact Energy Harvester Under Non-Smooth Forcing 

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## 1:1 Motion Under Triangle Wave

$$
\beta=\pi / 4
$$



The behavior under triangle wave forcing is similar to the behavior under harmonic forcing, with a slightly larger region of stable 1:1 solutions.

## The Model

Vibro-impact energy harvesters (VI-EH) convert vibrations into energy that can power small devices.




2:1 Motion

We model the ball's relative motion between impacts with a continuous differential equation.

$$
\dot{Z}(t)=-r \dot{Z}_{k}+\bar{g}\left(t-t_{k}\right)+F_{1}(t)+F_{1}\left(t_{k}\right)
$$

We calculate the impact velocities with maps.

$$
\begin{gathered}
P_{1}:\left(Z_{k} \in \partial B, \dot{Z}_{k}, t_{k}\right) \rightarrow\left(Z_{k+1} \in \partial T, \dot{Z}_{k+1}, t_{k+1}\right) \\
P_{2}:\left(Z_{k+1} \in \partial T, \dot{Z}_{k+1}, t_{k+1}\right) \rightarrow\left(Z_{k+2} \in \partial B, \dot{Z}_{k+2}, t_{k+2}\right)
\end{gathered}
$$

## Fourier Series Forcing

The qualitative behavior does not change as terms are added to the Fourier series. The location of the bifurcation varies slightly.


Terms in Fourier Series


## Our Project: Triangle Wave Forcing

Previous work assumed harmonic forcing. To more closely approximate experimental conditions, we study triangle wave forcing and Fourier approximations to this force.


2:1 Motion Under Triangle Wave

$$
\beta=\pi / 6
$$






Comparing against the harmonic wave, we notice a lack of grazing transition and a smaller range of stable solutions for the triangle wave.

## Conclusion and Next Steps

We conclude that the behavior under triangle wave forcing and intermediate Fourier approximations to this force is qualitatively similar to the behavior under harmonic forcing. The range of stable solutions tends to be larger for 1:1 motion, while the opposite tends to be true for $2: 1$ motion. We will continue to analyze similar behavior under varying parameters such as the angle of inclination and the restitution coefficient.

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