Analyzing the Energy Output of a Vibroimpacting System

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Intro/ Problem

Energy Harvesting (EH) has become a popular subject of discussion brought up by an ever-increasing want and need for clean renewable energy. In particular, the analysis of natural and man-made vibrations has become one of the forefronts of this discussion in recent years. With low cost, high energy outputs, and the ability to make small devices for everyday use, dielectric elastomer generators have become a focus of vibrational EH [1].

One such generator consists of:

- A canister of mass M with two membranes covering the ends
- Dielectric elastomer to produce capacitance
- A ball of mass m contained inside with $M \gg m$
- Inclined by angle $\beta$ away from horizontal in periodic motion

The question is, how do we harvest the most energy using this system?

Figure 1: Sketch of dielectric membrane based VI system. [2]

Main Question

How does the restitution constant, r, affect the system when equal to and less than one?

Equations

Using Newton’s laws of motion,

- Acceleration of the canister is set equal to a forcing function, arbitrarily set to cosine, over M. We name the canister displacement X.
- The ball experiences acceleration due to gravity, g, depending on $\beta$. We name the ball displacement x.
- Impact conditions involve restitution constant, r, which determines how elastic the collisions are.
- Represented by $\tau$, non-dimensional versions are created with the substitutions below [3]:

$$X(\tau) = x_c \cdot X^*(\tau), \quad Y(\tau) = y_c \cdot Y^*(\tau), \quad \tau = t_v \cdot t_c,$$

$$\hat{X}(\tau) = \frac{\omega \tau + \varphi}{M}, \quad \hat{Y}(\tau) = F(\tau) + \varphi = f(t),$$

$$\ddot{X}(\tau) = -\frac{MG}{||F||}, \quad \ddot{Y}(\tau) = -g = -g,$$

$$\dddot{x} = -r\dddot{x} + (r+1)\dddot{X}.$$ 

With displacement, velocity, and time constants of appropriate units:

$$x_c = \frac{\|\hat{F}\| \pi}{\omega M}, \quad y_c = \frac{\|\hat{F}\| \pi}{\omega M}, \quad t_c = \frac{\pi}{\omega}.$$ 

The non-dimensional distance of the canister is $d = x / s$, being the dimensional length of the model [3]. We use the non-dimensional equations to simplify calculations. After some equation manipulation we find explicit expressions for relative velocity, impact times, and phase [3]. Each expression contains the other two variables (i.e. velocity is found by an equation containing impact times and phase variables and so on). We then proceed to solve for the values by varying the non-dimensional length of the canister inside of a MATLAB solver.

From top to bottom, $r = 0.25, 0.5, 0.75,$ and 1.0 with all graphs having $\beta = \pi/3, ||F|| = 5N, M = 0.1245 \text{ kg}, g = 9.8 \text{ m/s}^2, \omega = \pi, U_{in} = 2000 \text{ F}$, and $f_0 \sim [1.19: 0.008: 3.6] \text{ Hz}$. The red line indicates the average energy output at each impact in the model within a given time interval. The black lines indicate the individual energy harvested at each impact.

Results

Discussion

Since the non-dimensional length is based on an inverse relationship with $\tau$, the length increases with frequency. Thus, we have multiple non-dimensional lengths for varying frequencies. Some interesting behaviors visible in the graphs include the chaotic movement with hundreds of points crammed together, areas where period doubling occurs, and the different areas in which the ball exhibits different types of periodic motion. In both the second and third graphs at $d = 2$, there are clearly three black branches, indicating that the ball is hitting the bottom of the canister twice before hitting the top in every period until the larger $d$-values. Afterwards, the ball is in a state of chaos and finally settles down into two round shaped groupings with two prongs each leading to a single path. These two groupings show us period doubling where there is still repeating periodic motion, but there are two paths taken for a cycle to be completed. The ball then settles into periodic motion with one impact at the bottom and top per period and a considerably high energy output. It seems as though increasing the restitution constant increases the energy output up until a certain point, in which the energy output starts to decay.

Future Work

One thing to note is that when $r = 1$, there is a rather sharp change in average power output. This has been shown in [1] in greater detail as well. Another future exploration could be figuring out why the sudden drop off happens and if there is any way to correct it with said r.

Under certain conditions, the ball would bounce off a membrane only to graze the same membrane before colliding with the other in periodic motion. This grazing ends up having little to no energy harvested as it barely makes contact with the membrane. One possible road to follow down is looking into what causes grazing phenomena and if a system must have grazing phenomena in its motion, what are the optimal parameters for harvesting maximal energy?

Figure 6: This is an example of grazing. The red line represents the path that the ball travels inside the capsule while the blue lines represent the motion of the capsule.

References


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