

Markov Chain Modeling of Inelastic Impacts in Energy Harvesters

 $F_k = \text{matrix from } k^{th} \text{ side to } k + 1^{th}$

Probability Transition Matrix = $L_k \circ F_k \circ L_{k-1} \circ F_{k-1}$

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

 $L_k = \text{impact matrix on } k^{th} \text{ side}$



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Approach:

Markov

Chain

 \dot{Z}^+ = relative velocity between ball and capsule after impact

 \dot{Z}^- = relative velocity between ball and capsule before impact

 $\Delta t = \text{time between impact}$

r = restitution coefficient (bounciness)

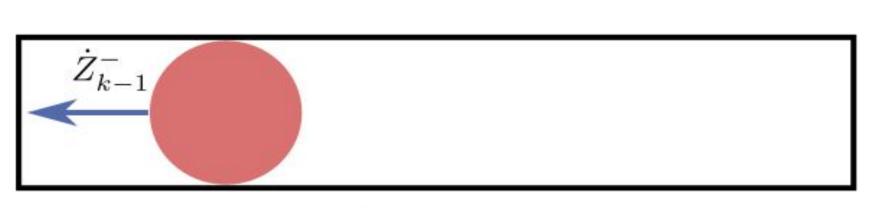
Question:

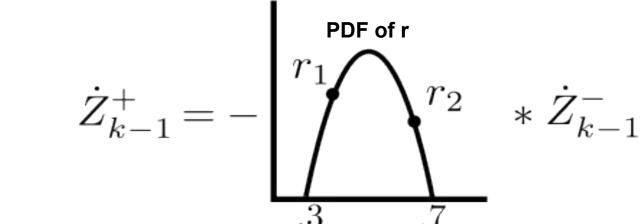
Does \dot{Z}^+ have predictable long-term behavior?

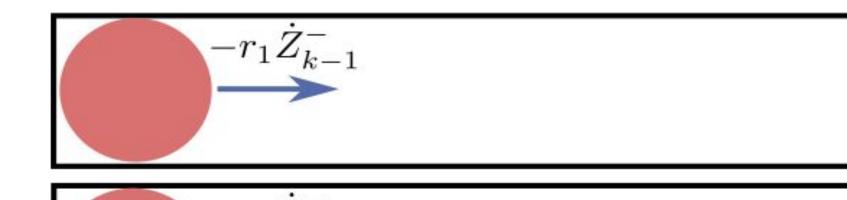
$$\ddot{Z}_t = \cos(\pi \Delta t + \phi_{k-1}) + \bar{g}$$

$$\dot{Z}_{t} = \frac{1}{\pi} (\sin(\pi \Delta t + \phi_{k-1}) - \sin(\phi_{k-1})) + \bar{g} \Delta t + \dot{Z}_{t_{k-1}}^{+}$$

$$Z_t = -\frac{1}{\pi^2} (\cos(\pi \Delta t + \phi_{k-1}) - \cos(\phi_{k-1})) - \frac{1}{\pi} \sin(\phi_{k-1}) \Delta t + \bar{g} \frac{\Delta t^2}{2} + \dot{Z}_{t_{k-1}}^+ \Delta t + d/2$$





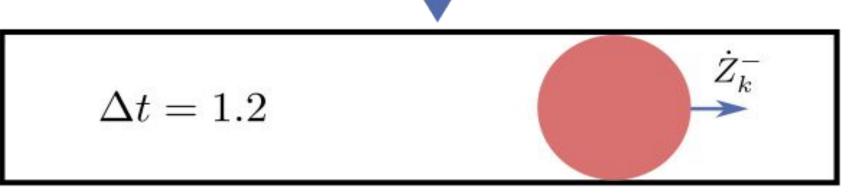


Problem 1: $-r_2Z_k^-$

Cosine

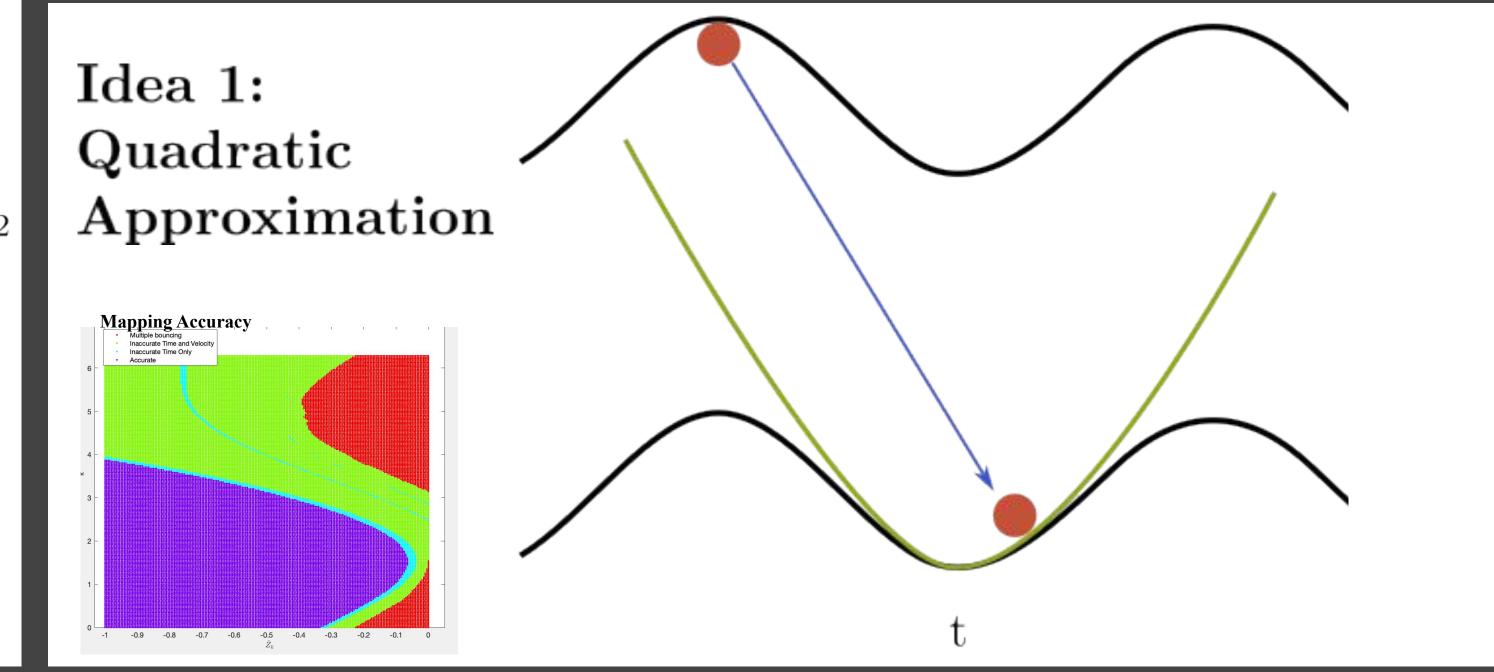
Dynamics

 $-d/2 = -\frac{1}{\pi^2}(\cos(\pi\Delta t + \phi_{k-1}) - \cos(\phi_{k-1})) - \frac{1}{\pi}\sin(\phi_{k-1})\Delta t + \dot{Z}_{k-1}^+ \Delta t + d/2$

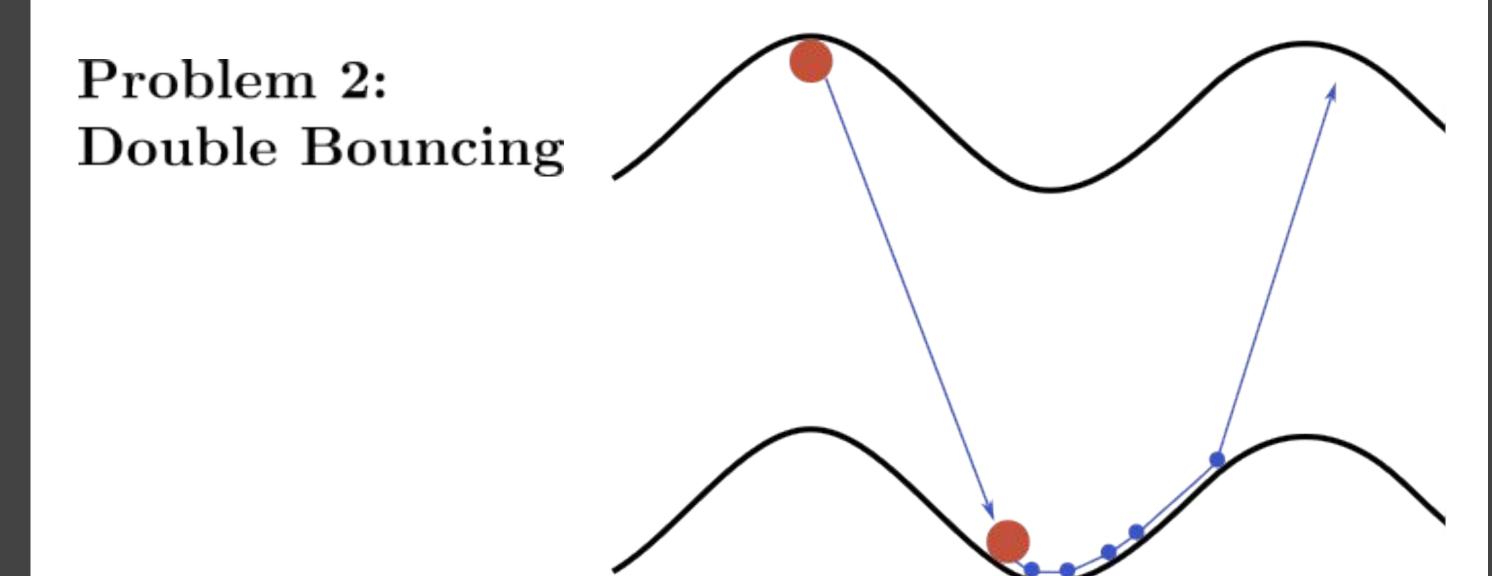


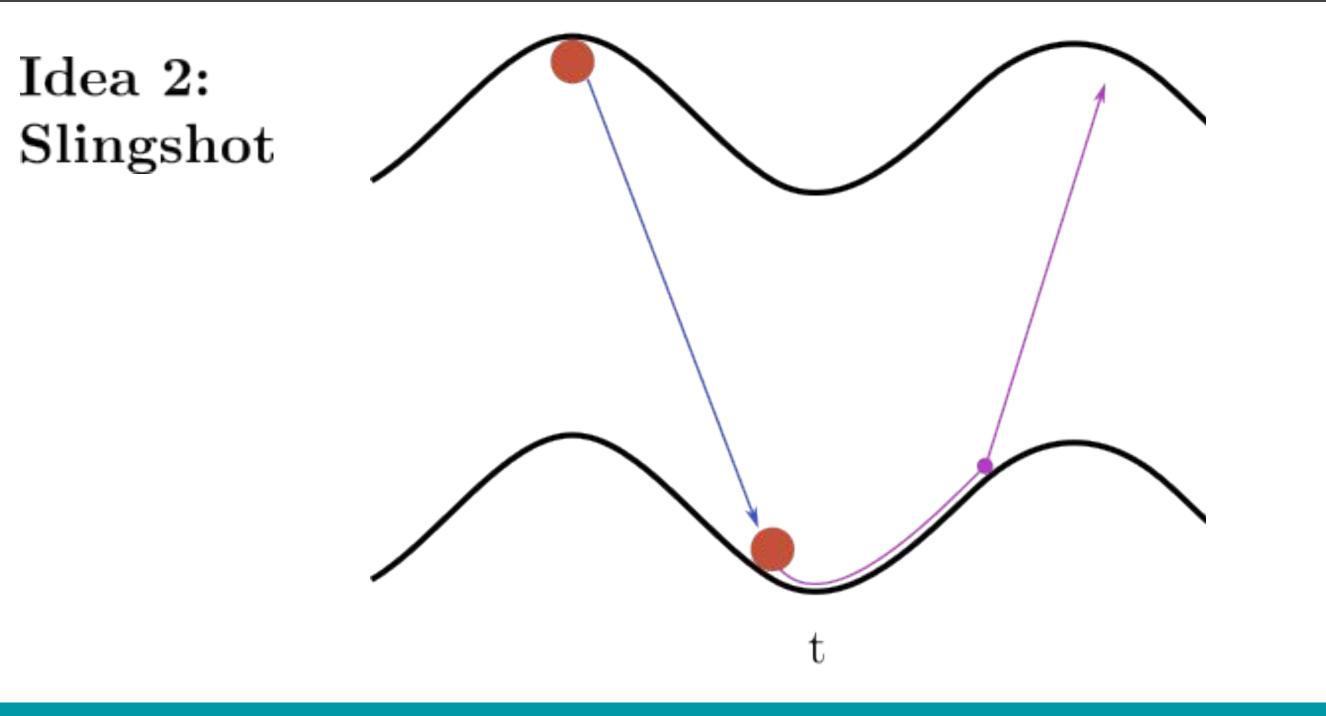


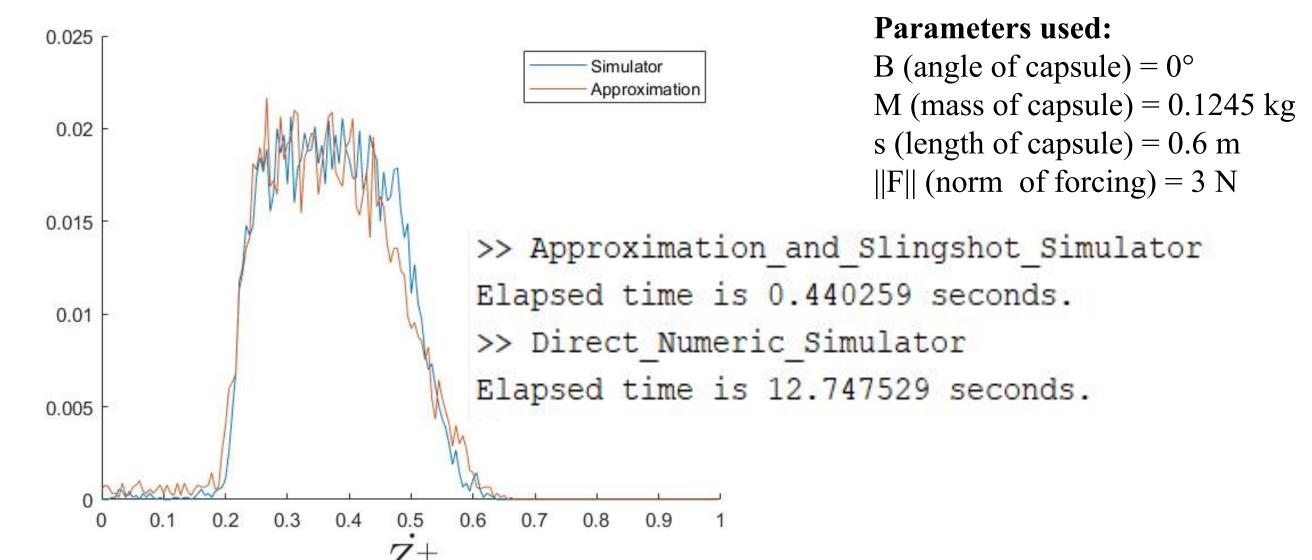
F_{k-1}

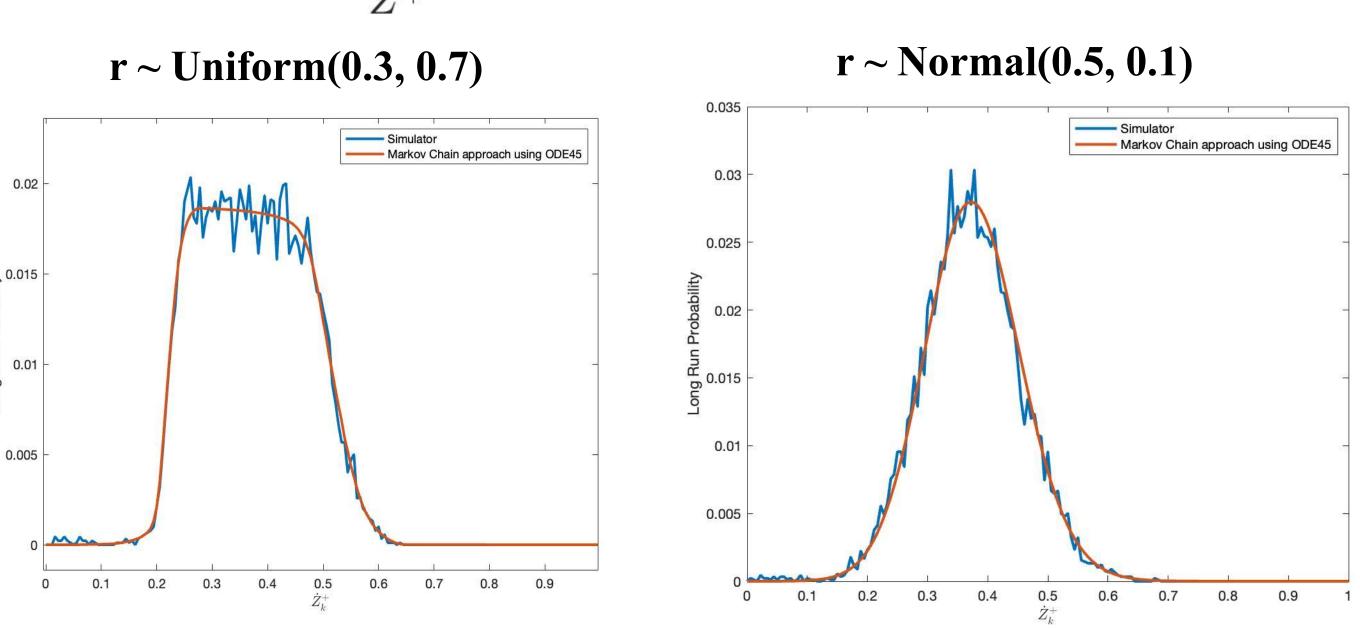


0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

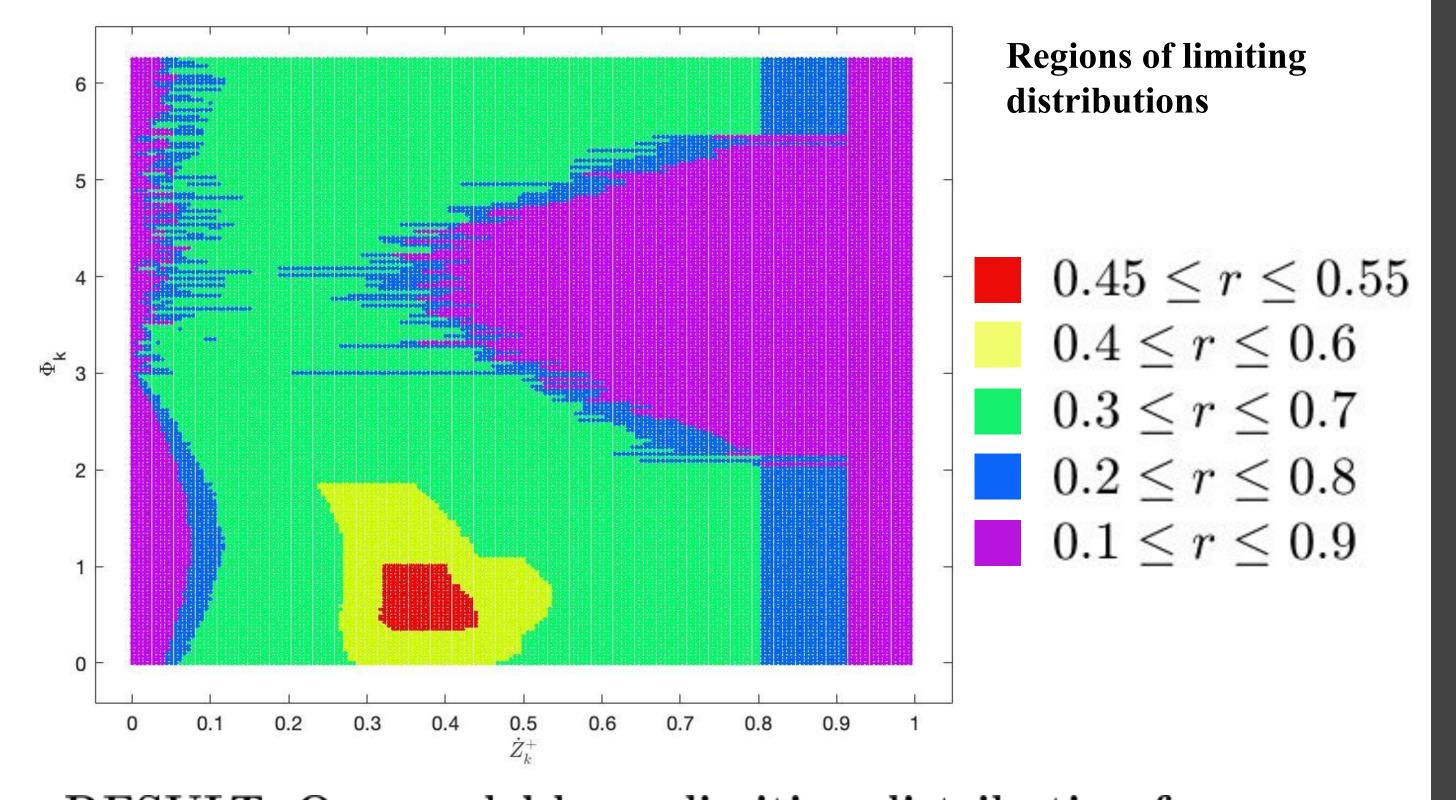








BIG IDEA: If a range R_1 of r has a limiting distribution for \dot{Z}^+ and $R_1 \subseteq R_2$ then R_2 has a limiting distribution for \dot{Z}^+ .



RESULT: Our model has a limiting distribution for capsule lengths $\in [0.3:0.05:0.75]$ and $r \sim uniform[0.45, 0.55]$

Conclusion

Using our model, we can find varying limiting distributions for \dot{Z}^+ under a wide range of parameters. In realistic scenarios, simulations suggest the existence of limiting distributions, and our model qualitatively matches those simulations.

Acknowledgements

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