

Solving System of Polynomials via Analytic Continuation and Monodromy

Timothy Cheek¹ and Taoran Wen²; mentored by Anton Leykin²



¹University of Michigan and ²Georgia Institute of Technology

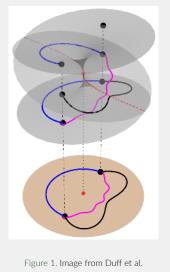
Main Goal

For generic $f_1(\vec{p}; \vec{x}), \ldots, f_n(\vec{p}; \vec{x}) \in \mathbb{C}[\vec{p}, \vec{x}]$, we aim to solve the square system

$$F_{\vec{p}}(\vec{x}) = \begin{bmatrix} f_1(\vec{p}; \vec{x}) \\ \vdots \\ f_n(\vec{p}; \vec{x}) \end{bmatrix} = 0.$$

Previous Solvers

Track solutions (homotopy continuation) around branch points (monodromy):



Problem:

Numerical instability of path trackers near branch points.

Analytic Continuation

Used to follow solution around branch points (instead of numerical ODE solvers).

$Im(log(z)) \xrightarrow{7}_{0} \xrightarrow{-2}_{Re(z)} \xrightarrow{7}_{4-4} \xrightarrow{7}_{4-4}$

Figure 2. Image from Makoto Yamashita

Padé Approximation

Used to capture behavior of variety around fixed parameter; is the "best" approximation. Also, poles of this rational function line up with branch points.

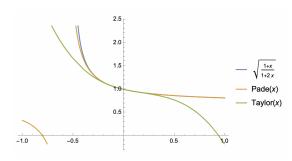


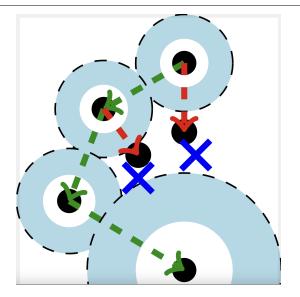
Figure 3. Example of Padé being closer approximation than Taylor for same degree

Extension of Homotopy Continuation

Same set up of

 $H(\vec{x},t) = F_{t\vec{p}_0+(1-t)\vec{p}_1} = 0$ but take $t \in \mathbb{C}$ rather than $t \in [0,1]$ to allow for flexibility in path (to keep safe distance from branch points).

Algorithm



Success

With mild assumptions, algorithm demonstrates competitive accuracy compared to state-of-the-art solvers.