# Counting Filling Pairs on Surfaces 

## Ellie Cirillo ${ }^{1}$, Jiamin Li${ }^{2}$, Alice Ponte ${ }^{3}$

Mentors: Dr. Wade Bloomquist ${ }^{3}$ and Dr. Dan Margalit ${ }^{3}$

## Filling Curves on a Surface

Two curves are filling if they cut the surface into a collection of disks.

If a pair of filling curves intersects minimally, it cuts the surface into a single disk.


## Main Question

How many distinct filling pairs of minimally intersecting curves are on a genus g surface?
$n(g)=$ \# distinct filling pairs

Building Surfaces with Filling Pairs of Curves


## Theorem

$$
n(3)=12 \quad n(4)=672
$$

Technique: pairs of curves $\sim$ permutations
Note: only 8 pairs are decomposable.

Upper Bound for Genus g Surface
$2^{2 g-2}(4 g-5)(2 g-3)!-2(2 g-1)\left[2 \cdot 2^{2 g-4} \cdot(2 g\right.$ $\left.-4)!+(2(2 g-1)-6)^{2} \cdot 2^{2 g-5}(2 g-5)!\right]$

Distance in the Curve Graph


Palaparthi - Mahanta (2021)
$(a, b)$ filling pair $\rightarrow d\left(a, T_{b}(a)\right)=4$

## Corollary

We have new examples of distance 4 curves.

Thank you to Georgia Tech for the opportunity, and to the NSF and SSP.

