Counting Filling Pairs on Surfaces
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Filling Curves on a Surface
- Two curves are \textit{filling} if they cut the surface into a collection of disks.
- If a pair of filling curves intersects \textit{minimally}, it cuts the surface into a \textit{single} disk.

Filling Pair on the Torus

Building Surfaces with Filling Pairs of Curves

Theorem
\begin{align*}
n(3) &= 12 \quad n(4) = 672
\end{align*}

Technique: pairs of curves \sim permutations

Note: only 8 pairs are \textit{decomposable}.

Upper Bound for Genus \( g \) Surface
\begin{align*}
2^{2g-2}(4g -5)(2g- 3)! - 2(2g - 1) [2 \cdot 2^{2g-4} \cdot (2g - 4)! + (2(2g - 1) - 6) \cdot 2^{2g-5}(2g - 5)!]
\end{align*}

Distance in the Curve Graph
\begin{align*}
\text{Palaparthi - Mahanta (2021)} \quad (a,b) \text{ filling pair} \rightarrow d(a,T_b(a)) = 4
\end{align*}

Corollary
We have new examples of distance 4 curves.

Main Question
How many distinct filling pairs of minimally intersecting curves are on a genus \( g \) surface?

\[ n(g) = \# \text{ distinct filling pairs} \]

Aougab and Huang Construction

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