## Knot: A Math Talk

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## What is a Knot?

A knot is a potentially knotted circle in space.


What is a Braid?


The Braid Group

$$
B_{n}=\left\langle\sigma_{1}, \ldots, \sigma_{n-1} \left\lvert\, \begin{array}{l}
\sigma_{i} \sigma_{j}=\sigma_{j} \sigma_{i} \text { if }|i-j| \nmid \\
\sigma_{i} \sigma_{j} \sigma_{i}=\sigma_{j} \sigma_{i} \sigma_{j} \text { if }|i-j|=1
\end{array}\right.\right\rangle
$$

How are Knots and Braids Related?


## Can any knot be obtained from abraid?

Theorem [Alexander]: Every knot can be realized as the closure of some braid.


## When do two braids make the same knot?

## Theorem [Markov]:

Suppose two braid words $\beta 1$ and $\beta 2$ have the same closure. Then $\beta 1$ and $\beta 2$ are related by

- Braid relations
- Conjugation
- De-stabilization


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## Far Relation

Close Relation

- Conjugation
- De-stabilization


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Racetrack

- Conjugation
- De-stabilization



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De-stabilization

- Conjugation
- De-stabilization




## Braid Index

The braid index is the least number of strands needed to make a knot.

(P,Q) Torus Knots

$$
\begin{aligned}
& \text { \# of strands } \\
& i(T(p, q))=\min \{p, q\}= \begin{cases}p & p<q \\
q & q<p\end{cases}
\end{aligned}
$$

$T(4,3)$


## 1-Bridge Braido K(w,B,T)



Goal: What is the Braid Index of 1-Bridge Braids?

## How do we calculate the braid index?

A full twist on n strands: $\left(\sigma_{n-1} \ldots \sigma_{1}\right)^{n}$

## Theorem (Morton; Franks-Williams):

If $\beta$ is a positive braid word on $n$ strands, and $\beta \approx \omega *$ (full twist), then $i(\beta)=n$.


## Theorem $[G-K-L-N-T-W]:$

The braid index of any 1 -bridge braid $K(w, b, t)$ is:


## Proof Strategy

- Apply Markov Moves
- Find a full-twist
- Use MFW Theorem



## Why Care about 1-Bridge Braids?

Knots/links can be used to build any 3-manifold by doing Dehn Surgery!

This involves:

- drilling out a knotted tube
- gluing a solid torus back in



## Why Care about 1-Bridge Braids?

## Theorem (Gabai '89):

If Dehn surgery along a knot in the solid torus produces a solid torus, then the knot is either a torus knot or a 1-bridge braid.


Questions?

