$$A_j(x) = a_j \cos\left(\frac{j\pi}{L} x\right)$$

$$B_j(t) = e^{-k \frac{j^2\pi^2}{L^2} t} \qquad \left(\lambda_j = -k \frac{j^2\pi^2}{L^2}\right)$$

$$\boxed{u_j(x,t) = a_j \cos\left(\frac{j\pi}{L} x\right) e^{-k \frac{j^2\pi^2}{L^2} t}}$$

$$\begin{cases} \dfrac{\partial u_j}{\partial t} = k \dfrac{\partial^2 u_j}{\partial x^2} \\ \dfrac{\partial u_j}{\partial x}(0,t) = 0 = \dfrac{\partial u_j}{\partial x}(L,t) \end{cases}$$

$$u_j(x,0) \stackrel{?}{=} g(x) \quad (??)$$

[redacted]

CASE 2 $\lambda = 0$.

$$A'' = 0, \implies A(x) = ax + b$$

$$\underline{A'(0) = 0 = A'(L)} \longrightarrow A' = a = 0.$$

$A_0 = a_0$ (const.)

$\underline{B' = 0}$

$u_0(x,t) = a_0$ (const.)

↑ interesting function

[redacted]

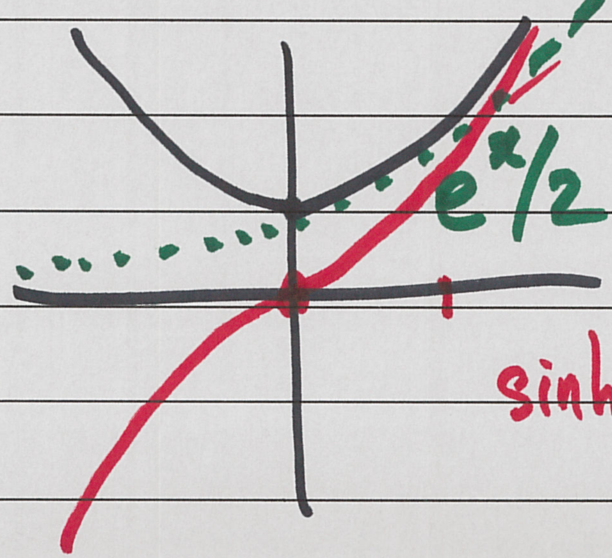
CASE 3 $A'' = \frac{\lambda}{k} A \ , \quad \frac{\lambda}{k} > 0.$

$\qquad\qquad\quad \overset{\parallel}{\mu^2}, \ \mu > 0$

x not t!

general solution: $A(x) = a\, e^{-\mu x} + b\, e^{\mu x}$

or $A(x) = a \cosh \mu x + b \sinh \mu x$



$\cosh x = \dfrac{e^{-x} + e^{x}}{2}$

$\sinh x = \dfrac{e^{x} - e^{-x}}{2}$

[redacted]

$$A(x) = a \cosh \mu x + b \sinh \mu x$$

$$A'(x) = a\mu \sinh \mu x + b\mu \cosh \mu x$$

$$A'(0) = b\mu = 0 \implies b = 0.$$

$$\implies A(x) = a \cosh \mu x$$

$$A'(L) = a\mu \sinh(\underline{\underline{\mu L}}) \neq 0.$$

CASE 3 gives no solution

separated variables

[redacted]

separated variables "solutions"

$u_0 = a_0$ (CASE 2)

$u_j = a_j \cos\left(\frac{j\pi}{L} x\right) e^{-k \frac{j^2\pi^2}{L^2} t}$

$j = 1, 2, 3, \ldots$

$u_t = k\, u_{xx}$

$u_x(0,t) = 0 = u_x(L,t)$

Try a superposition:

$$u(x,t) = \underbrace{\sum_{j=0}^{\infty} a_j e^{-k \frac{j^2\pi^2}{L^2} t} \cos\left(\frac{j\pi}{L} x\right)}$$

Can this solve my problem (original)?

[redacted]

Only condition I'm worried about:

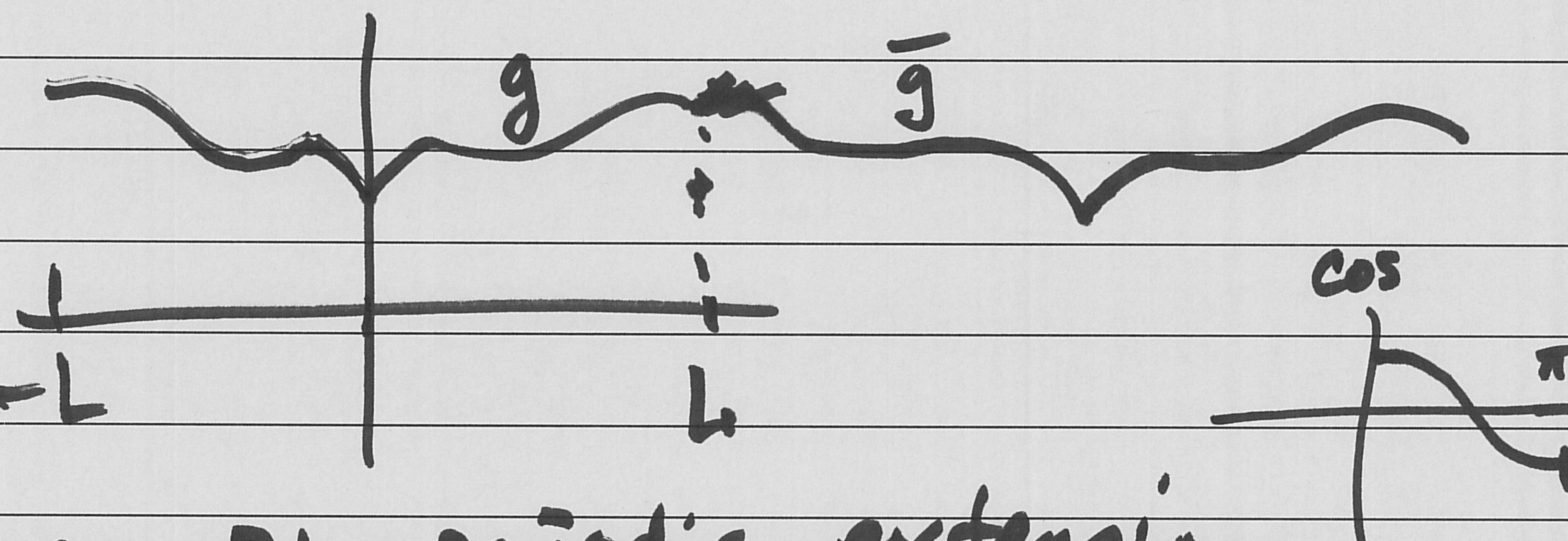
$$u(x,0) = \sum_{j=0}^{\infty} a_j \cos \frac{j\pi}{L} x = \underline{\underline{g(x)}}$$

Can we pick the a_j's so this is true?

<u>Fourier's Question.</u>

[redacted]

Expansion of g in a Fourier cosine series



even $2L$-periodic extension.

$$\bar{g}(x) = \sum_{j=0}^{\infty} a_j \cos \frac{j\pi}{L} x$$

$$\boxed{\frac{1}{L}\int_0^L g(x)\,dx = a_0}$$

[redacted]

$$\bar{g}(x) = \sum_{j=0}^{\infty} a_j \cos \frac{j\pi}{L} x$$

$$\boxed{a_k \overset{?}{=} \frac{2}{L} \int_0^L g(x) \cos \frac{k\pi}{L} x}$$

$$\underbrace{\int_{-L}^{L} \bar{g} \cos \frac{k\pi}{L} x \, dx}_{2 \int_0^{L} g(x) \cos \frac{k\pi}{L} x \, dx} = \sum_{j=0}^{\infty} a_j \int_{-L}^{L} \cos \frac{j\pi}{L} x \cos \frac{k\pi}{L} x \, dx$$

$$= a_k \int_{-L}^{L} \cos^2 \frac{k\pi}{L} x \, dx$$ [redacted]

$$L$$