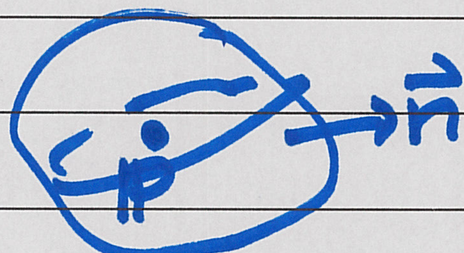
What is the divergence of a vector field?

BAD ANSWER

$$\mathrm{div}\, \mathbb{V} = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3}$$

GOOD ANSWER

LOOK AT $\int_{\partial R} \mathbb{V} \cdot \vec{n}$



imagine $\lim_{R \to \{\mathbb{P}\}} \int_{\partial R} \mathbb{V} \cdot \vec{n}$

[redacted]

$$\lim_{R \to \{\mathbb{P}\}} \int_{\partial R} \mathbb{V} \cdot \vec{n} = 0$$

$$\sum \underbrace{\mathbb{V}(\vec{x}_j^*) \cdot \vec{n}(\vec{x}_j^*)}_{\leq |\mathbb{V}|} A_j$$

$\uparrow$ little areas

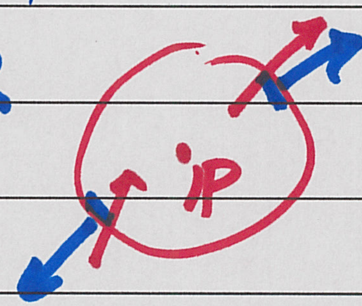
TRY

$$\lim_{R \to \{\mathbb{P}\}} \frac{1}{\mu(\partial R)} \int_{\partial R} \mathbb{V} \cdot \vec{n} = 0$$

$\uparrow$ area of ∂R

$$\sum A_j = \text{Area}(\partial R)$$

$$\sim \frac{0}{0}$$



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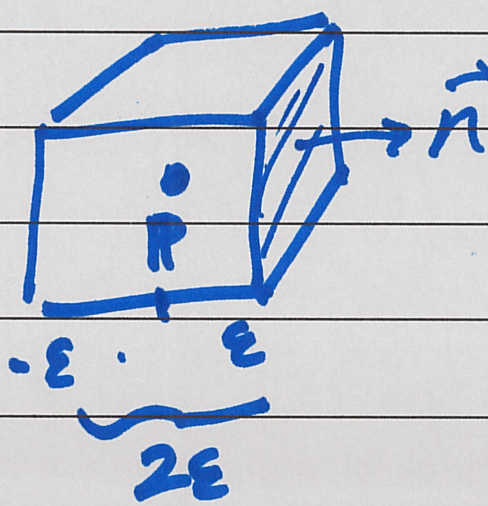
Good Answer

$$\boxed{\operatorname{div} \mathbb{V}(\mathbb{P}) = \lim_{R \to \{\mathbb{P}\}} \frac{1}{\mu(R)} \int_{\partial R} \mathbb{V} \cdot \vec{n}}$$

volume (n-dim'l measure) of R

Exercise:



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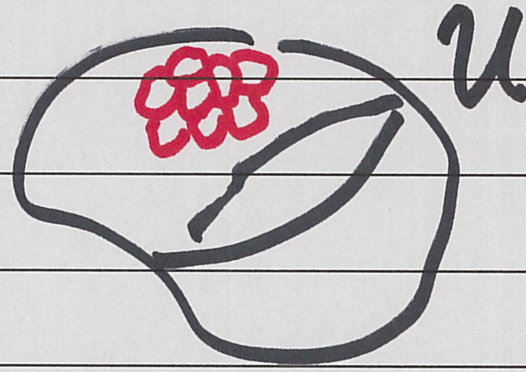
$$\operatorname{div} \mathbb{V}(P) = \lim_{R \to \{P\}} \frac{1}{\operatorname{vol}(R)} \int_{\partial R} \mathbb{V} \cdot \vec{n}$$

"PROOF" of The DIVERGENCE THM:



$\mathcal{U}$

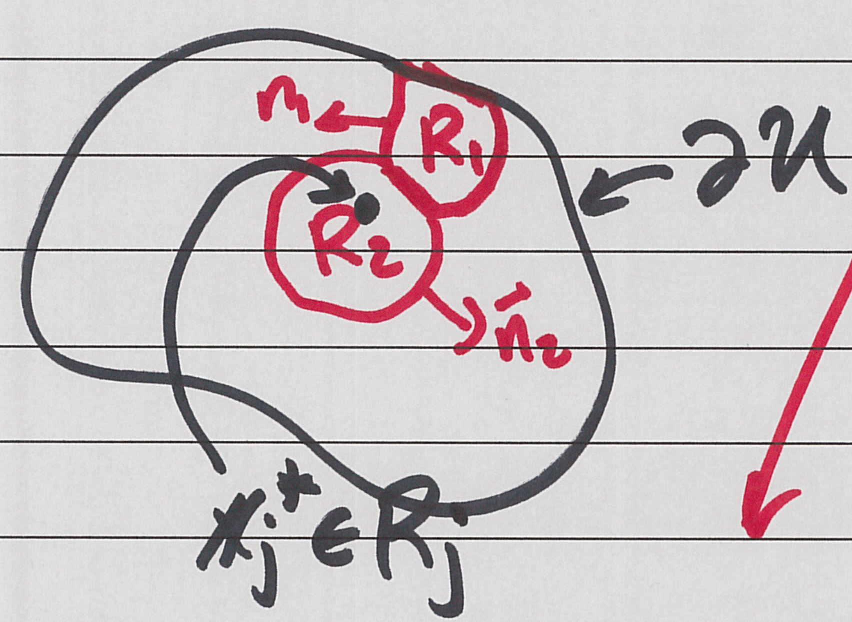
$$\int_{\mathcal{U}} \operatorname{div} \mathbb{V} = \int_{\partial \mathcal{U}} \mathbb{V} \cdot \vec{n}$$

cut $\mathcal{U}$ up into little regions

R_j (not overlapping - nice)

[redacted]

$$\int_{\partial U} \mathbb{V} \cdot \vec{n} = \sum_j \int_{\partial R_j} \mathbb{V} \cdot n_j \qquad (?)$$



$$\operatorname{div} \mathbb{V}(x_j^*) \sim \frac{1}{\operatorname{vol}(R_j)} \int_{\partial R_j} \mathbb{V} \cdot \vec{n}_j$$

$$\sim \sum_j \operatorname{div} \mathbb{V}(x_j^*) \operatorname{vol}(R_j)$$

$$\to \int_U d\mathbb{V}$$

[redacted]