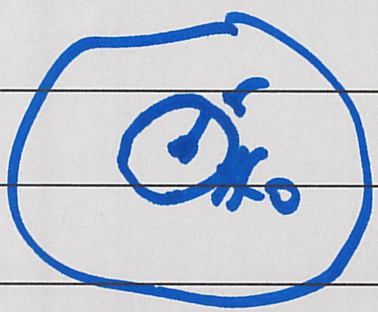
Assume $f(x_0) > 0$. Then there is some $r > 0$ so that

$$f(x) > \frac{f(x_0)}{2} \quad \text{if} \quad \underline{|x - x_0| < r}.$$

Now take $\phi \geq 0$ with supp ϕ inside

$$B_r(x_0) = \{x \in U : |x - x_0| < r\} \quad \underline{\text{AND}} \quad \phi(x_0) > 0$$

"ball"



U

$$0 = \int_U f\phi = \int_{B_r(x_0)} f\phi \geq \frac{f(x_0)}{2}\int\phi > 0.$$

$\times$

[redacted]

<u>Conclusion</u>: If u is a minimizer of

$$\mathcal{J}[u] = \int_U F(x, u, Du)$$

then u satisfies the PDE

$$\boxed{\frac{\partial F}{\partial z}(x,u,Du) - \sum_{j=1}^{n} \frac{\partial}{\partial x_j}\left(\frac{\partial F}{\partial p_j}(x,u,Du)\right) = 0}$$

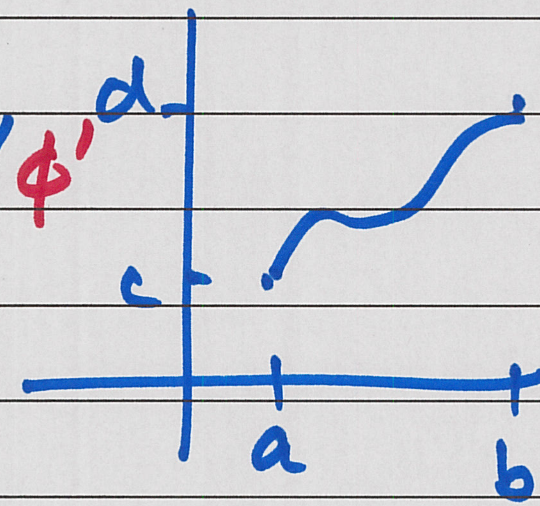
↑ Euler-Lagrange PDE
for $\mathcal{J}$.

[redacted]

Examples:

$$\ell[u] = \int_a^b \sqrt{1+u'(x)^2}\, dx$$

$$\delta \ell_u[\phi] = \int_a^b \frac{1}{2}\left(1+u'^2\right)^{-1/2} \cdot 2u'\phi'$$



$u+\varepsilon\phi$

$$\sqrt{1+(u'+\varepsilon\phi')^2}$$

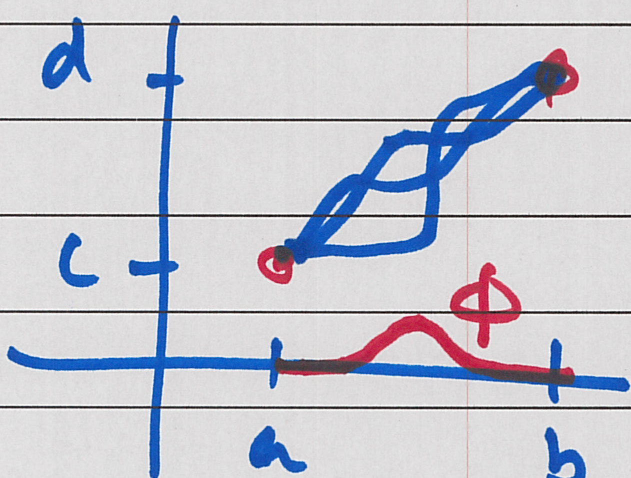
$$F = F(p) = \sqrt{1+p^2}$$

$$\frac{\partial F}{\partial p} = \frac{1}{2}\left(1+p^2\right)^{-1/2} \cdot 2p$$

Integrate by parts:

$$0 = \int_a^b \underbrace{\frac{u'}{\sqrt{1+u'^2}}}_{u} \underbrace{\phi'}_{dv} = \frac{u'}{\sqrt{1+u'^2}}\phi\Big|_a^b - \int_a^b \left(\frac{u'}{\sqrt{1+u'^2}}\right)'\phi$$

$\phi(a) = \phi(b) = 0$



$$\left(\frac{u'}{\sqrt{1+u'^2}}\right)' = 0$$

curvature of graph(u)

$$\frac{u''}{\sqrt{1+u'^2}} - \frac{u'^2}{(1+u'^2)^{3/2}} \cdot u'' = \frac{u''}{(1+u'^2)^{3/2}}$$

[redacted]