MATH 4581 Lecture 17 Thursday October 21, 2021

TODAY I'M EXCITED
ABOUT THE WAVE EQUATION.
($u_{tt} = \sigma^2 u_{xx}$)

LAST TIME WAS A BIT OF A DISASTER.

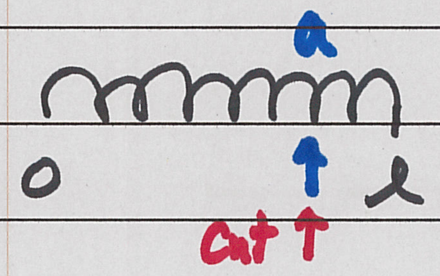
Fourier's Theorem * Pointwise Convergence

Integral Convergence in L^2 (Hilbert space)

Eigenfunction expansion
$$\begin{cases} u_t = u_{xx} \\ u(0,t) = t \\ u(L,t) = t \\ u(x,0) = 0 \end{cases}$$

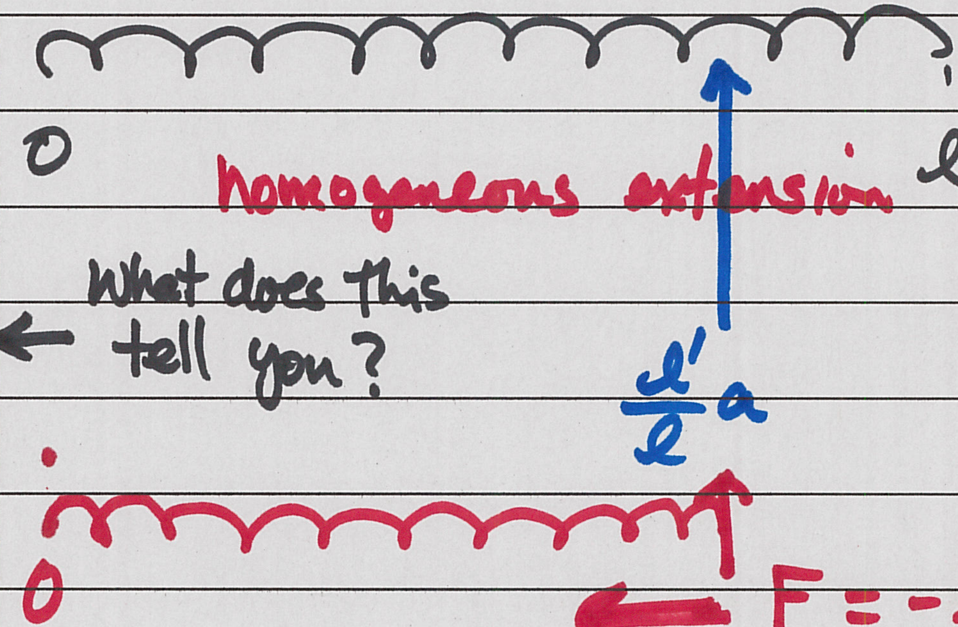
[redacted]

Spring



Hooke's constant

$F = -k(l' - l)$



$F = -k(l' - l)$

$$\boxed{k\left(\frac{l'}{l}a - a\right) = k\frac{a}{l}(l' - l)}$$

What does this tell you?

length of extension is

$$\boxed{\frac{l'}{l}a - a} = a\left(\frac{l'}{l} - 1\right) = \frac{a}{l}(l' - l)$$

[redacted]

Hooke's constant is not materially determined.

Hooke's constant depends on the material/kind of spring <u>and</u> <u>also</u> the length.

<u>EXTENSION</u> $\underline{\frac{l'}{l} a - a}$, <u>Force $-k(l'-l)$</u>

What is the new Hooke's constant k_a for the piece of spring with equilibrium length $a < l$.

$$\boxed{k_a = \frac{l}{a} k}$$ $k_a a = kl$ [redacted]

ELASTICITY ϵ

What is the physical significance of this force?

Eigenfunction Expansion:

$$W_t = W_{xx} - 1$$

$$W_t = W_{xx} - \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{2k+1} \sin\left(\frac{(2k+1)\pi}{L} x\right)$$

$$A_k(x) B_k(0) = 0$$

$$(W_k)_t = (W_k)_{xx} - \frac{4}{\pi} \cdot \frac{1}{2k+1} \underbrace{\sin\left(\frac{(2k+1)\pi}{L} x\right)}$$

$\sin\left(\frac{(2k+1)\pi}{L} x\right)$ is an eigenfunction

the ~~best~~ operator $\underline{\underline{W_{xx}}}$

INITIAL COND.

$W_k = A_k(x) B_k(t)$ gives

$$B_k' + \frac{(2k+1)^2 \pi^2}{L^2} B_k = -\frac{4}{(2k+1)\pi}$$

[redacted]

Riesz-Fischer Theorem:

There is a space of functions called L^2

$$\int_I f^2 < \infty$$

Hilbert: This space has an inner product:

$$\langle f, g \rangle_{L^2} = \int_I fg$$

$$\left| \int_I fg \right| \leq \sqrt{\int f^2}\sqrt{\int g^2}$$

$$d(f,g) = \sqrt{\int (f-g)^2}$$

[redacted]

Riesz-Fischer Theorem: The Fourier series of every L^2 function converges to the function in L^2.

$$\|f_K - f\|_{L^2} = \sqrt{\int_I (f_K - f)^2} \xrightarrow{K \nearrow \infty} 0$$

$$\left(f_K = \sum_{j=1}^{K} a_j \sin\left(\frac{j\pi}{L} x\right) \right)$$

$$a_j = \frac{2}{L} \int f(x) \sin\left(\frac{j\pi}{L} x\right) dx$$

[redacted]

Inner products

Inner product space. } vector spaces
normed space
metric space
topological space

$\|\cdot\| : X \longrightarrow [0,\infty)$ norm

(i) $\|x\| = 0 \iff x = 0$ ← zero vector

(ii) $\|cx\| = |c|\,\|x\|$ ← scaling.

(iii) $\|x+y\| \leq \|x\| + \|y\|$

$d(p,q) \leq d(p,z) + d(z,q)$

$d(x,y) = \|x-y\|$ norm induced distance

[redacted]

$$\langle \cdot , \cdot \rangle : X \times X \to \mathbb{R}$$

(i) $\langle x, x \rangle \geq 0$ with "=" if and only if $x = \phi$.

(ii) $\langle x, y \rangle = \langle y, x \rangle$

(iii) $\langle ax + by, z \rangle = a\langle x, z \rangle + b \langle y, z \rangle$

bilinearity

Inner product

$\|x\| = \sqrt{\langle x, x \rangle}$ induced norm

$d(x,y) = \|x-y\| = \sqrt{\langle x-y, x-y \rangle}$

[redacted]